# A-Level Mathematics

# Year 12 Transition Booklet

## Contents

**Introduction to Mathematics A-Level**

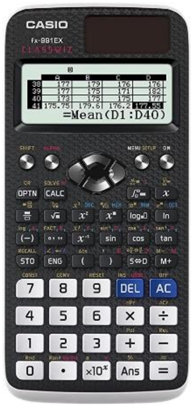
Mathematics at A-Level is an enjoyable, cognitively demanding course and a highly valued qualification. The course consists of three interconnected strands; Pure Mathematics, Statistics and Mechanics (Applied Mathematics)

Examination Board: Edexcel
Course Code: 9MA0

| | |
|---|---|
| **Paper 1: Pure Mathematics**<br>**33%**<br>**2 hours**<br>**100 marks** | **Any pure content can be assessed on either paper** |
| **Paper 2: Pure Mathematics**<br>**33%**<br>**2 hours**<br>**100 marks** | |
| **Paper 3: Mechanics and Statistics**<br>**33%**<br>**2 hours**<br>**100 marks** | All assessments are sat at the end of Year 13 |

Textbook: You will be loaned copies of the key course textbooks in September.

Calculator: The required calculator for this course is the Casio ClassWiz FX-991EX advanced scientific calculator which has all the functionality required by the exam board. This can be purchased from the school via Parent Pay or purchased independently, if you wish.



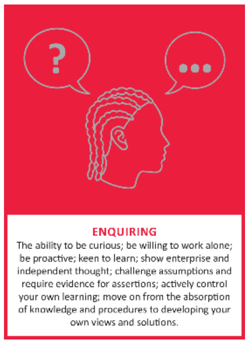

Higher resolution calculators are permitted, but not necessary. **You must have a calculator by September.**

Independent Study:

There is an expectation that you complete 10 hours of independent study per week for A-Level Mathematics. This includes the time you spend on your homework. Guidance on how to spend your independent study time will be given in September.


Useful Resources:
https://www.adamsmaths.uk/home/as-maths
https://www.physicsandmathstutor.com/
https://www.tayyubmajeed.com/home
https://www.drfrostmaths.com/

Support: Each topic in the course has an accompanying 'Owen Video' to help consolidate new learning. Regular support sessions will be held throughout the course from the Autumn Term.




Stretch: (Useful Websites)
https://nrich.maths.org/post-16
https://undergroundmathematics.org/

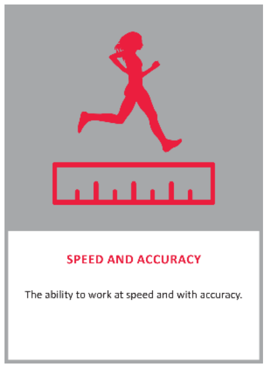
Teachers:

Mr Akhtar taakhtar@SMCS.org.uk
Miss Zaman szaman@SMCS.org.uk

**Summer Transition Tasks**

Success in A-Level Mathematics firstly relies on having strong foundational GCSE knowledge on key topics including;

- Indices,
- Surds,
- Algebra,
- Solving Simultaneous Equations,
- Straight line graphs,
- Quadratics,
- Trigonometry,




To support you in your transition to A-Level we have provided a set of **compulsory** tasks for you to complete over the Summer break. This is to be handed in during your first lesson in September.

Please write on the booklet and mark your answers in a red or green pen. Attach any additional sheets to the transition booklet.

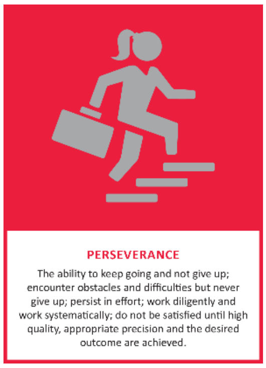
Managing your time:

Do not try to do all of the booklet in one go. Spaced revision will be most effective. A suggested timetable to help you structure your revision is below:
Support:

You have been directed to Hegarty Maths Clips for each section of the booklet. Please also refer to your GCSE notes/resources for support. If you find that you need additional help on any of the topics, please let your Maths teacher know and we will provide opportunities for support in September.

As you work through this booklet you should make a note on this checklist of where you needed help. If you are still unsure about a topic, tick the final column.

Please do not just pretend you are ok with these topics if you are struggling! We are here to help! We will put on extra sessions to help you sort out these problems early on in the course.




| Week | TOPIC | Exercise | HM Clip | How did it go? | | |
|---|---|---|---|---|---|---|
| | | | | 🙂 | 😐 | ☹️ |
| w/c 18th July | Removing brackets | A | 160-164 | | | |
| | | B | 165-166 | | | |
| | Linear equations | A | 184- 185 | | | |
| | | B | 186 | | | |
| | | C | 187 | | | |
| w/c 25th July | Simultaneous equations | A | 190-195 | | | |
| w/c 1st August | Factorising | A | 168-169 | | | |
| | | B | 223-228 | | | |
| w/c 8th August | Change the subject of the formula | A | 285 | | | |
| | | B | 286 | | | |
| | | C | 287 | | | |
| w/c 15th August | Solving quadratic equations | A | 230-234 | | | |
| w/c 22nd August | Indices | A | 103-110 | | | |
| | | B | | | | |
| | Completing the Square | A | 235-239 | | | |
| w/c 29th August | Practice Test | | | | | |

HM Clips: 160-166

# Chapter 1: REMOVING BRACKETS

To remove a single bracket, we multiply every term in the bracket by the number or the expression on the outside:

**Examples**

1) $3(x + 2y) = 3x + 6y$

2) $-2(2x - 3) = (-2)(2x) + (-2)(-3)$
$= -4x + 6$

To expand two brackets, we must multiply everything in the first bracket by everything in the second bracket. We can do this in a variety of ways, including

* the smiley face method
* FOIL (Fronts Outers Inners Lasts)
* using a grid.

**Examples**:

1) $(x + 1)(x + 2) = x(x + 2) + 1(x + 2)$

or

$(x + 1)(x + 2) = x^2 + 2 + 2x + x$
$= x^2 + 3x + 2$

or

| | $x$ | 1 |
|---|---|---|
| $x$ | $x^2$ | $x$ |
| 2 | $2x$ | 2 |

$(x + 1)(x + 2) = x^2 + 2x + x + 2$
$= x^2 + 3x + 2$

2) $(x - 2)(2x + 3) = x(2x + 3) - 2(2x + 3)$
$= 2x^2 + 3x - 4x - 6$
$= 2x^2 - x - 6$

or

$(x - 2)(2x + 3) = 2x^2 - 6 + 3x - 4x = 2x^2 - x - 6$

or

| | $x$ | -2 |
|---|---|---|
| $2x$ | $2x^2$ | $-4x$ |
| 3 | $3x$ | -6 |

$(2x + 3)(x - 2) = 2x^2 + 3x - 4x - 6$
$= 2x^2 - x - 6$

**EXERCISE A** Multiply out the following brackets and simplify.

1. $7(4x + 5)$
2. $-3(5x - 7)$
3. $5a - 4(3a - 1)$
4. $4y + y(2 + 3y)$
5. $-3x - (x + 4)$
6. $5(2x - 1) - (3x - 4)$
7. $(x + 2)(x + 3)$
8. $(t - 5)(t - 2)$
9. $(2x + 3y)(3x - 4y)$
10. $4(x - 2)(x + 3)$
11. $(2y - 1)(2y + 1)$
12. $(3 + 5x)(4 - x)$

**Two Special Cases**

| **Perfect Square:** | **Difference of two squares:** |
|---|---|
| $(x + a)^2 = (x + a)(x + a) = x^2 + 2ax + a^2$ | $(x - a)(x + a) = x^2 - a^2$ |
| $(2x - 3)^2 = (2x - 3)(2x - 3) = 4x^2 - 12x + 9$ | $(x - 3)(x + 3) = x^2 - 3^2$ |
| | $= x^2 - 9$ |

**EXERCISE B** Multiply out

1. $(x - 1)^2$
2. $(3x + 5)^2$
3. $(7x - 2)^2$
4. $(x + 2)(x - 2)$
5. $(3x + 1)(3x - 1)$
6. $(5y - 3)(5y + 3)$

HM Clips: 184-186

# Chapter 2: LINEAR EQUATIONS

When solving an equation, you must remember that whatever you do to one side must also be done to the other. You are therefore allowed to

- add the same amount to both side
- subtract the same amount from each side
- multiply the whole of each side by the same amount
- divide the whole of each side by the same amount.

If the equation has unknowns on both sides, you should collect all the letters onto the same side of the equation.

If the equation contains brackets, you should start by expanding the brackets.

A linear equation is an equation that contains numbers and terms in $x$. A linear equation does not contain any $x^2$ or $x^3$ terms.

**More help** on solving equations can be obtained by downloading the leaflet available at this website: http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-simplelinear.pdf

**Example 1**: Solve the equation $64 - 3x = 25$

**Solution**: There are various ways to solve this equation. One approach is as follows:

Step 1: Add $3x$ to both sides (so that the $x$ term is positive): $64 = 3x + 25$

Step 2: Subtract 25 from both sides: $39 = 3x$

Step 3: Divide both sides by 3: $13 = x$

So the solution is $x = 13$.

**Example 2**: Solve the equation $6x + 7 = 5 - 2x$.

**Solution:**

Step 1: Begin by adding $2x$ to both sides (to ensure that the $x$ terms are together on the same side) $8x + 7 = 5$

Step 2: Subtract 7 from each side: $8x = -2$

Step 3: Divide each side by 8: $x = -\frac{1}{4}$

**Exercise A**: Solve the following equations, showing each step in your working:

1) $2x + 5 = 19$  2) $5x - 2 = 13$  3) $11 - 4x = 5$

4) $5 - 7x = -9$  5) $11 + 3x = 8 - 2x$  6) $7x + 2 = 4x - 5$

**Example 3**: Solve the equation $2(3x - 2) = 20 - 3(x + 2)$

Step 1: Multiply out the brackets: $6x - 4 = 20 - 3x - 6$
(taking care of the negative signs)

Step 2: Simplify the right hand side: $6x - 4 = 14 - 3x$

Step 3: Add 3x to each side: $9x - 4 = 14$

Step 4: Add 4: $9x = 18$

Step 5: Divide by 9: $x = 2$

**Exercise B:** Solve the following equations.

1) $5(2x - 4) = 4$

2) $4(2 - x) = 3(x - 9)$

3) $8 - (x + 3) = 4$

4) $14 - 3(2x + 3) = 2$

HM Clips: 187

**EQUATIONS CONTAINING FRACTIONS**

When an equation contains a fraction, the first step is usually to multiply through by the denominator of the fraction. This ensures that there are no fractions in the equation.

**Example 4**: Solve the equation $\frac{y}{2} + 5 = 11$

**Solution**:
Step 1: Multiply through by 2 (the denominator in the fraction): $y + 10 = 22$

Step 2: Subtract 10: $y = 12$

**Example 5**: Solve the equation $\frac{1}{3}(2x + 1) = 5$

**Solution**:
Step 1: Multiply by 3 (to remove the fraction) $2x + 1 = 15$

Step 2: Subtract 1 from each side $2x = 14$

Step 3: Divide by 2 $x = 7$

When an equation contains two fractions, you need to multiply by the lowest common denominator. This will then remove both fractions.

**Example 6**: Solve the equation $\frac{x+1}{4} + \frac{x+2}{5} = 2$

**Solution**:
Step 1: Find the lowest common denominator: The smallest number that both 4 and 5 divide into is 20.

Step 2: Multiply both sides by the lowest common denominator $\frac{20(x+1)}{4} + \frac{20(x+2)}{5} = 40$

Step 3: Simplify the left hand side: $\frac{\overset{5}{\cancel{20}}(x+1)}{\cancel{4}} + \frac{\overset{4}{\cancel{20}}(x+2)}{\cancel{5}} = 40$

$5(x + 1) + 4(x + 2) = 40$

Step 4: Multiply out the brackets: $5x + 5 + 4x + 8 = 40$

Step 5: Simplify the equation: $9x + 13 = 40$

Step 6: Subtract 13 $9x = 27$

Step 7: Divide by 9: $x = 3$

**Example 7**: Solve the equation $x + \frac{x-2}{4} = 2 - \frac{3-5x}{6}$

**Solution**: The lowest number that 4 and 6 go into is 12. So we multiply every term by 12:

$$12x + \frac{12(x-2)}{4} = 24 - \frac{12(3-5x)}{6}$$

Simplify $12x + 3(x-2) = 24 - 2(3-5x)$

Expand brackets $12x + 3x - 6 = 24 - 6 + 10x$

Simplify $15x - 6 = 18 + 10x$

Subtract $10x$ $5x - 6 = 18$

Add 6 $5x = 24$

Divide by 5 $x = 4.8$

**Exercise C**: Solve these equations

1) $\frac{1}{2}(x+3) = 5$

2) $\frac{2x}{3} - 1 = \frac{x}{3} + 4$

3) $\frac{y}{4} + 3 = 5 - \frac{y}{3}$

4) $\frac{x-2}{7} = 2 + \frac{3-x}{14}$

5) $\frac{7x-1}{2} = 13 - x$

6) $\frac{y-1}{2} + \frac{y+1}{3} = \frac{2y+5}{6}$

7) $2x+\frac{x-1}{2}=\frac{5x+3}{3}$

8) $2-\frac{5}{x}=\frac{10}{x}-1$

HM Clips: 190-195

**Chapter 3: SIMULTANEOUS EQUATIONS**

An example of a pair of simultaneous equations is $3x + 2y = 8$ ○
$5x + y = 11$ ○

In these equations, $x$ and $y$ stand for two numbers. We can solve these equations in order to find the values of $x$ and $y$ by eliminating one of the letters from the equations.

In these equations it is simplest to eliminate $y$. We do this by making the coefficients of $y$ the same in both equations. This can be achieved by multiplying equation ○ by 2, so that both equations contain $2y$:

$3x + 2y = 8$ ○
$10x + 2y = 22$ 2×○ = ○

To eliminate the $y$ terms, we subtract equation ○ from equation ○. We get: $7x = 14$
i.e. $x = 2$

To find y, we substitute $x = 2$ into one of the original equations. For example if we put it into ○:

$10 + y = 11$
$y = 1$

Therefore the solution is $x = 2$, $y = 1$.

**Remember**: You can <u>check</u> your solutions by substituting both x and y into the original equations.

**Example**: Solve $2x + 5y = 16$ ○
$3x - 4y = 1$ ○

**Solution**: We begin by getting the same number of $x$ or $y$ appearing in both equation. We can get $20y$ in both equations if we multiply the top equation by 4 and the bottom equation by 5:

$8x + 20y = 64$ ○
$15x - 20y = 5$ ○

As the SIGNS in front of $20y$ are DIFFERENT, we can eliminate the $y$ terms from the equations by ADDING:

$23x = 69$ ○+○
i.e. $x = 3$

Substituting this into equation ○ gives:

$6 + 5y = 16$
$5y = 10$
So… $y = 2$
The solution is $x = 3$, $y = 2$.

If you need **more help** on solving simultaneous equations, you can download a booklet from the following website:
http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-simultaneous1.pdf

**Exercise A**:

Solve the pairs of simultaneous equations in the following questions:

1) $x + 2y = 7$
$3x + 2y = 9$

2) $x + 3y = 0$
$3x + 2y = -7$

3) $3x - 2y = 4$
$2x + 3y = -6$

4) $9x - 2y = 25$
$4x - 5y = 7$

5) $4a + 3b = 22$
$5a - 4b = 43$

6) $3p + 3q = 15$
$2p + 5q = 14$

## Chapter 4: FACTORISING

### Common factors

We can factorise some expressions by taking out a common factor.

**Example 1**: Factorise $12x - 30$

**Solution**: 6 is a common factor to both 12 and 30. We can therefore factorise by taking 6 outside a bracket:

$$12x - 30 = 6(2x - 5)$$

**Example 2**: Factorise $6x^2 - 2xy$

**Solution**: 2 is a common factor to both 6 and 2. Both terms also contain an $x$.
So we factorise by taking $2x$ outside a bracket.

$$6x^2 - 2xy = 2x(3x - y)$$

**Example 3**: Factorise $9x^3y^2 - 18x^2y$

**Solution**: 9 is a common factor to both 9 and 18.
The highest power of $x$ that is present in both expressions is $x^2$.
There is also a $y$ present in both parts.
So we factorise by taking $9x^2y$ outside a bracket:

$$9x^3y^2 - 18x^2y = 9x^2y(xy - 2)$$

**Example 4**: Factorise $3x(2x - 1) - 4(2x - 1)$

**Solution**: There is a common bracket as a factor.
So we factorise by taking $(2x - 1)$ out as a factor.
The expression factorises to $(2x - 1)(3x - 4)$

### Exercise A

Factorise each of the following

1) $3x + xy$

2) $4x^2 - 2xy$

3) $pq^2 - p^2q$

4) $3pq - 9q^2$

5) $2x^3 - 6x^2$

6) $8a^5b^2 - 12a^3b^4$

7) $5y(y - 1) + 3(y - 1)$

HM Clips: 223-228

**Factorising quadratics**

**Simple quadratics: Factorising quadratics of the form** $x^2 + bx + c$
The method is:
Step 1: Form two brackets $(x \ \ldots \ )(x \ \ldots \ )$
Step 2: Find two numbers that multiply to give $c$ and add to make $b$. These two numbers get written at the other end of the brackets.

**Example 1**: Factorise $x^2 - 9x - 10$.

**Solution**: We need to find two numbers that multiply to make -10 and add to make -9. These numbers are -10 and 1.
Therefore $x^2 - 9x - 10 = (x - 10)(x + 1)$.

**General quadratics: Factorising quadratics of the form** $ax^2 + bx + c$
The method is:
Step 1: Find two numbers that multiply together to make $ac$ and add to make $b$.
Step 2: Split up the $bx$ term using the numbers found in step 1.
Step 3: Factorise the front and back pair of expressions as fully as possible.
Step 4: There should be a common bracket. Take this out as a common factor.

**Example 2**: Factorise $6x^2 + x - 12$.

**Solution**: We need to find two numbers that multiply to make $6 \times -12 = -72$ and add to make 1. These two numbers are -8 and 9.

Therefore, $6x^2 + x - 12 = \underbrace{6x^2 - 8x} + \underbrace{9x - 12}$

$= 2x(3x - 4) + 3(3x - 4)$ (the two brackets must be identical)

$= (3x - 4)(2x + 3)$

**Difference of two squares: Factorising quadratics of the form** $x^2 - a^2$

Remember that $x^2 - a^2 = (x + a)(x - a)$.

Therefore: $x^2 - 9 = x^2 - 3^2 = (x+3)(x-3)$
$16x^2 - 25 = (2x)^2 - 5^2 = (2x+5)(2x-5)$

Also notice that: $2x^2 - 8 = 2(x^2 - 4) = 2(x+4)(x-4)$
and $3x^3 - 48xy^2 = 3x(x^2 - 16y^2) = 3x(x+4y)(x-4y)$

**Factorising by pairing**
We can factorise expressions like $2x^2 + xy - 2x - y$ using the method of factorising by pairing:

$2x^2 + xy - 2x - y = x(2x + y) - 1(2x + y)$ (factorise front and back pairs, ensuring both brackets are identical)
$= (2x + y)(x - 1)$

If you need **more help** with factorising, you can download a booklet from this website:
http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-factorisingquadratics.pdf

**Exercise B**

Factorise

1) $x^2 - x - 6$

2) $x^2 + 6x - 16$

3) $2x^2 + 5x + 2$

4) $2x^2 - 3x$ (factorise by taking out a common factor)

5) $3x^2 + 5x - 2$

6) $2y^2 + 17y + 21$

7) $7y^2 - 10y + 3$

8) $10x^2 + 5x - 30$

9) $4x^2 - 25$

10) $x^2 - 3x - xy + 3y^2$

11) $4x^2 - 12x + 8$

12) $16m^2 - 81n^2$

13) $4y^3 - 9a^2 y$

14) $8(x+1)^2 - 2(x+1) - 10$

## Chapter 5: CHANGING THE SUBJECT OF A FORMULA

We can use algebra to change the subject of a formula. Rearranging a formula is similar to solving an equation – we must do the same to both sides in order to keep the equation balanced.

**Example 1**: Make $x$ the subject of the formula $y = 4x + 3$.

**Solution**: $y = 4x + 3$

Subtract 3 from both sides: $y - 3 = 4x$

Divide both sides by 4; $\frac{y-3}{4} = x$

So $x = \frac{y-3}{4}$ is the same equation but with $x$ the subject.

**Example 2**: Make $x$ the subject of $y = 2 - 5x$

**Solution**: Notice that in this formula the $x$ term is negative.

$y = 2 - 5x$

Add $5x$ to both sides $y + 5x = 2$ (the $x$ term is now positive)

Subtract $y$ from both sides $5x = 2 - y$

Divide both sides by 5 $x = \frac{2-y}{5}$

**Example 3**: The formula $C = \frac{5(F-32)}{9}$ is used to convert between ° Fahrenheit and ° Celsius.

We can rearrange to make $F$ the subject.

$C = \frac{5(F-32)}{9}$

Multiply by 9 $9C = 5(F-32)$ (this removes the fraction)

Expand the brackets $9C = 5F - 160$

Add 160 to both sides $9C + 160 = 5F$

Divide both sides by 5 $\frac{9C+160}{5} = F$

Therefore the required rearrangement is $F = \frac{9C+160}{5}$.

**Exercise A**

Make $x$ the subject of each of these formulae:

1) $y = 7x - 1$

2) $y = \frac{x+5}{4}$

3) $4y = \frac{x}{3} - 2$

4) $y = \frac{4(3x - 5)}{9}$

**Rearranging equations involving squares and square roots**

**Example 4**: Make $x$ the subject of $x^2 + y^2 = w^2$

**Solution**: $x^2 + y^2 = w^2$

Subtract $y^2$ from both sides: $x^2 = w^2 - y^2$ (this isolates the term involving $x$)

Square root both sides: $x = \pm\sqrt{w^2 - y^2}$

Remember that you can have a positive or a negative square root. We cannot simplify the answer any more.

**Example 5**: Make $a$ the subject of the formula $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

**Solution**: $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

Multiply by 4 $4t = \sqrt{\frac{5a}{h}}$

Square both sides $16t^2 = \frac{5a}{h}$

Multiply by $h$: $16t^2 h = 5a$

Divide by 5: $\frac{16t^2 h}{5} = a$

**Exercise B:**

Make $t$ the subject of each of the following

1) $P = \frac{wt}{32r}$

2) $P = \frac{wt^2}{32r}$

3) $V = \frac{1}{3}\pi t^2 h$

4) $P = \sqrt{\frac{2t}{g}}$

5) $Pa = \frac{w(v-t)}{g}$

6) $r = a + bt^2$

**More difficult examples**

Sometimes the variable that we wish to make the subject occurs in more than one place in the formula. In these questions, we collect the terms involving this variable on one side of the equation, and we put the other terms on the opposite side.

**Example 6**: Make $t$ the subject of the formula $a - xt = b + yt$

**Solution**: $a - xt = b + yt$

Start by collecting all the t terms on the right hand side:

Add $xt$ to both sides: $a = b + yt + xt$

Now put the terms without a $t$ on the left hand side:

Subtract $b$ from both sides: $a - b = yt + xt$

Factorise the RHS: $a - b = t(y + x)$

Divide by $(y + x)$: $\frac{a-b}{y+x} = t$

So the required equation is $t = \frac{a-b}{y+x}$

**Example 7**: Make $W$ the subject of the formula $T - W = \frac{Wa}{2b}$

**Solution**: This formula is complicated by the fractional term. We begin by removing the fraction:

Multiply by $2b$: $2bT - 2bW = Wa$

Add $2bW$ to both sides: $2bT = Wa + 2bW$ (this collects the W's together)

Factorise the RHS: $2bT = W(a + 2b)$

Divide both sides by $a + 2b$: $W = \frac{2bT}{a+2b}$

If you need more help you can download an information booklet on rearranging equations from the following website:

http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-formulae2-tom.pdf

**Exercise C**

Make $x$ the subject of these formulae:

1) $ax + 3 = bx + c$

2) $3(x + a) = k(x - 2)$

3) $y = \dfrac{2x+3}{5x-2}$

4) $\dfrac{x}{a} = 1 + \dfrac{x}{b}$

HM Clips: 230-234, 241, 242

## Chapter 6: SOLVING QUADRATIC EQUATIONS

A quadratic equation has the form $ax^2 + bx + c = 0$.

There are two methods that are commonly used for solving quadratic equations:
* factorising
* the quadratic formula

Note that not all quadratic equations can be solved by factorising. The quadratic formula can always be used however.

### Method 1: Factorising

Make sure that the equation is rearranged so that the right hand side is 0. It usually makes it easier if the coefficient of $x^2$ is positive.

**Example 1** : Solve $x^2 - 3x + 2 = 0$

Factorise $(x - 1)(x - 2) = 0$
Either $(x - 1) = 0$ or $(x - 2) = 0$
So the solutions are $x = 1$ or $x = 2$

Note: The individual values $x = 1$ and $x = 2$ are called the **roots** of the equation.

**Example 2**: Solve $x^2 - 2x = 0$

Factorise: $x(x - 2) = 0$
Either $x = 0$ or $(x - 2) = 0$
So $x = 0$ or $x = 2$

### Method 2: Using the formula

Recall that the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Example 3**: Solve the equation $2x^2 - 5 = 7 - 3x$

**Solution**: First we rearrange so that the right hand side is 0. We get $2x^2 + 3x - 12 = 0$
We can then tell that $a = 2$, $b = 3$ and $c = -12$.
Substituting these into the quadratic formula gives:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-12)}}{2 \times 2} = \frac{-3 \pm \sqrt{105}}{4}$$ (this is the *surd form* for the solutions)

If we have a calculator, we can evaluate these roots to get: $x = 1.81$ or $x = -3.31$

If you need more help with the work in this chapter, there is an information booklet downloadable from this web site:
http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-quadraticequations.pdf

**EXERCISE A**

1) Use factorisation to solve the following equations:

a) $x^2 + 3x + 2 = 0$

b) $x^2 - 3x - 4 = 0$

c) $x^2 = 15 - 2x$

2) Find the roots of the following equations:

a) $x^2 + 3x = 0$

b) $x^2 - 4x = 0$

c) $4 - x^2 = 0$

3) Solve the following equations either by factorising or by using the formula:

a) $6x^2 - 5x - 4 = 0$

b) $8x^2 - 24x + 10 = 0$

4) Use the formula to solve the following equations to 3 significant figures. Some of the equations can't be solved.

a) $x^2 + 7x + 9 = 0$

b) $6 + 3x = 8x^2$

c) $4x^2 - x - 7 = 0$

d) $x^2 - 3x + 18 = 0$

e) $3x^2 + 4x + 4 = 0$

f) $3x^2 = 13x - 16$

HM Clips: 230-234, 241, 242

## Chapter 7: INDICES

**Basic rules of indices**

$y^4$ means $y \times y \times y \times y$. 4 is called the **index** (plural: indices), **power** or **exponent** of $y$.

There are 3 basic rules of indices:

1) $a^m \times a^n = a^{m+n}$ e.g. $3^4 \times 3^5 = 3^9$
2) $a^m \div a^n = a^{m-n}$ e.g. $3^8 \times 3^6 = 3^2$
3) $(a^m)^n = a^{mn}$ e.g. $\left(3^2\right)^5 = 3^{10}$

**Further examples**

$y^4 \times 5y^3 = 5y^7$

$4a^3 \times 6a^2 = 24a^5$ (multiply the numbers and multiply the $a$'s)

$2c^2 \times \left(-3c^6\right) = -6c^8$ (multiply the numbers and multiply the $c$'s)

$24d^7 \div 3d^2 = \dfrac{24d^7}{3d^2} = 8d^5$ (divide the numbers and divide the $d$ terms i.e. by subtracting the powers)

**Exercise A**

Simplify the following:

1) $b \times 5b^5 =$ (Remember that $b = b^1$)

2) $3c^2 \times 2c^5 =$

3) $b^2c \times bc^3 =$

4) $2n^6 \times (-6n^2) =$

5) $8n^8 \div 2n^3 =$

6) $d^{11} \div d^9 =$

7) $\left(a^3\right)^2 =$

8) $\left(-d^4\right)^3 =$

**More complex powers**

**Zero index:**
Recall from GCSE that

$$a^0 = 1.$$

This result is true for any non-zero number *a*.

Therefore $5^0 = 1$ $\left(\frac{3}{4}\right)^0 = 1$ $(-5.2304)^0 = 1$

**Negative powers**

A power of -1 corresponds to the reciprocal of a number, i.e. $a^{-1} = \frac{1}{a}$

Therefore $5^{-1} = \frac{1}{5}$

$$0.25^{-1} = \frac{1}{0.25} = 4$$

$\left(\frac{4}{5}\right)^{-1} = \frac{5}{4}$ (you find the reciprocal of a fraction by swapping the top and bottom over)

This result can be extended to more general negative powers: $a^{-n} = \frac{1}{a^n}$.

This means:

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

$$\left(\frac{1}{4}\right)^{-2} = \left(\left(\frac{1}{4}\right)^{-1}\right)^2 = \left(\frac{4}{1}\right)^2 = 16$$

**Fractional powers:**

Fractional powers correspond to roots: $a^{1/2} = \sqrt{a}$ $a^{1/3} = \sqrt[3]{a}$ $a^{1/4} = \sqrt[4]{a}$

In general:

$$a^{1/n} = \sqrt[n]{a}$$

Therefore:

$8^{1/3} = \sqrt[3]{8} = 2$ $25^{1/2} = \sqrt{25} = 5$ $10000^{1/4} = \sqrt[4]{10000} = 10$

A more general fractional power can be dealt with in the following way: $a^{m/n} = \left(a^{1/n}\right)^m$

So $4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$

$$\left(\frac{8}{27}\right)^{2/3} = \left(\left(\frac{8}{27}\right)^{1/3}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\left(\frac{25}{36}\right)^{-3/2} = \left(\frac{36}{25}\right)^{3/2} = \left(\sqrt{\frac{36}{25}}\right)^3 = \left(\frac{6}{5}\right)^3 = \frac{216}{125}$$

**Exercise B:**

Find the value of:

1) $4^{1/2}$

2) $27^{1/3}$

3) $\left(\frac{1}{9}\right)^{1/2}$

4) $5^{-2}$

5) $18^{0}$

6) $7^{-1}$

7) $27^{2/3}$

8) $\left(\frac{2}{3}\right)^{-2}$

9) $8^{-2/3}$

10) $(0.04)^{1/2}$

11) $\left(\frac{8}{27}\right)^{2/3}$

12) $\left(\frac{1}{16}\right)^{-3/2}$

Simplify each of the following:

13) $2a^{1/2} \times 3a^{5/2}$

14) $x^{3} \times x^{-2}$

15) $\left(x^{2}y^{4}\right)^{1/2}$

# Chapter 8: COMPLETING THE SQUARE

HM Clips: 235-239

Formula for C.T.S:

$$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

Completing the square is used to write out a quadratic equation:

$$x^2 + 2bx + b^2 = (x + b)^2$$
$$x^2 - 2bx + b^2 = (x - b)^2$$

To complete the square of the function $x^2 + 2bx$ you need a further term $b^2$.

So the completed square form is

$$x^2 + 2bx = (x + b)^2 - b^2$$

Similarly

$$x^2 - 2bx = (x - b)^2 - b^2$$

**Example 1**:

Complete the square for the expression $x^2 + 8x$

$x^2 + 8x$

$= (x + 4)^2 - 4^2$

$= (x + 4)^2 - 16$

**Example 2**:

Complete the square for expressions

a) $x^2 + 12x$

$= (x - 6)^2 - 6^2$

$= (x - 6)^2 - 36$

b) $2x^2 - 10x$

$2(x^2 - 5x)$

$= 2\left[\left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2\right]$

$= 2\left(x - \frac{5}{2}\right)^2 - \frac{25}{2}$

**Exercise A:**

Complete the square for the expressions:

1. $x^2 + 4x$
2. $x^2 - 6x$
3. $x^2 - 16x$
4. $x^2 + x$
5. $x^2 - 14x$
6. $2x^2 + 16x$
7. $3x^2 - 24x$
8. $2x^2 - 4x$

9. $5x^2 + 20x$  10. $2x^2 - 5x$  11. $3x^2 + 9x$  12. $3x^2 - x$

## Practice Booklet Test

This is a sample test that covers all of the topics in this booklet. Please complete this to consolidate your revision.

**You may NOT use a calculator**

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1. Expand and simplify
   (a) $(2x + 3)(2x - 1)$ (b) $(a + 3)^2$ (c) $4x(3x - 2) - x(2x + 5)$

2. Factorise
   (a) $x^2 - 7x$ (b) $y^2 - 64$ (c) $2x^2 + 5x - 3$(d) $6t^2 - 13t + 5$

3. Simplify
   (a) $\dfrac{4x^3y}{8x^2y^3}$ (b) $\dfrac{3x+2}{3} + \dfrac{4x-1}{6}$

4. Solve the following equations
   (a) $\dfrac{h-1}{4} + \dfrac{3h}{5} = 4$ (b) $x^2 - 8x = 0$ (c) $p^2 + 4p = 12$

5. Write each of the following as single powers of $x$ and / y
   (a) $\dfrac{1}{x^4}$ (b) $(x^2y)^3$ (c) $\dfrac{x^5}{x^{-2}}$

6. Work out the values of the following, giving your answers as fractions
   (a) $4^{-2}$ (b) $10^0$ (c) $\left(\dfrac{8}{27}\right)^{\frac{1}{3}}$

7. Solve the simultaneous equations $3x - 5y = -11$
   $5x - 2y = 7$

8. Rearrange the following equations to make $x$ the subject
   (a) $v^2 = u^2 + 2ax$ (b) $V = \dfrac{1}{3}\pi x^2 h$(c) $y = \dfrac{x+2}{x+1}$

9. Solve $5x^2 - x - 1 = 0$ giving your solutions in surd form

10. If $x^2 + 6x + 4 = (x + p)^2 + q$
    Find the values of $p$ and $q$

## SOLUTIONS TO THE EXERCISES

### CHAPTER 1:

Ex A

1) $28x + 35$  2) $-15x + 21$  3) $-7a + 4$  4) $6y + 3y^2$  5) $2x - 4$
6) $7x - 1$  7) $x^2 + 5x + 6$  8) $t^2 - 3t - 10$  9) $6x^2 + xy - 12y^2$
10) $4x^2 + 4x - 24$  11) $4y^2 - 1$  12) $12 + 17x - 5x^2$

Ex B

1) $x^2 - 2x + 1$  2) $9x^2 + 30x + 25$  3) $49x^2 - 28x + 4$  4) $x^2 - 4$
5) $9x^2 - 1$  6) $25y^2 - 9$

### CHAPTER 2

Ex A

1) 7  2) 3  3) 1½  4) 2  5) -3/5  6) -7/3

Ex B

1) 2.4  2) 5  3) 1  4) ½

Ex C

1) 7  2) 15  3) 24/7  4) 35/3  5) 3  6) 2  7) 9/5  8) 5

Ex D

1) 34, 36, 38  2) 9.875, 29.625  3) 24, 48

### CHAPTER 3

1) $x = 1, y = 3$  2) $x = -3, y = 1$3) $x = 0, y = -2$4) $x = 3, y = 1$
5) $a = 7,\ b = -2$6) $p = 11/3,\ q = 4/3$

### CHAPTER 4

Ex A

1) $x(3 + y)$  2) $2x(2x - y)$  3) $pq(q - p)$  4) $3q(p - 3q)$  5) $2x^2(x - 3)$  6) $4a^3b^2(2a^2 - 3b^2)$
7) $(y - 1)(5y + 3)$

Ex B

1) $(x - 3)(x + 2)$  2) $(x + 8)(x - 2)$  3) $(2x + 1)(x + 2)$  4) $x(2x - 3)$  5) $(3x - 1)(x + 2)$
6) $(2y + 3)(y + 7)$  7) $(7y - 3)(y - 1)$  8) $5(2x - 3)(x + 2)$  9) $(2x + 5)(2x - 5)$  10) $(x - 3)(x - y)$
11) $4(x - 2)(x - 1)$  12) $(4m - 9n)(4m + 9n)$  13) $y(2y - 3a)(2y + 3a)$  14) $2(4x + 5)(x - 4)$

### CHAPTER 5

Ex A

1) $x = \dfrac{y+1}{7}$  2) $x = 4y - 5$  3) $x = 3(4y + 2)$  4) $x = \dfrac{9y + 20}{12}$

Ex B

1) $t = \dfrac{32rP}{w}$  2) $t = \pm\sqrt{\dfrac{32rP}{w}}$  3) $t = \pm\sqrt{\dfrac{3V}{\pi h}}$  4) $t = \dfrac{P^2 g}{2}$  5) $t = v - \dfrac{Pag}{w}$  6) $t = \pm\sqrt{\dfrac{r-a}{b}}$

Ex C

1) $x = \dfrac{c-3}{a-b}$  2) $x = \dfrac{3a + 2k}{k - 3}$  3) $x = \dfrac{2y + 3}{5y - 2}$  4) $x = \dfrac{ab}{b - a}$

### CHAPTER 6

1) a) -1, -2  b) -1, 4  c) -5, 3  2) a) 0, -3  b) 0, 4  c) 2, -2
3) a) -1/2, 4/3  b) 0.5, 2.5  4) a) -5.30, -1.70  b) 1.07, -0.699  c) -1.20, 1.45
d) no solutions  e) no solutions  f) no solutions

### CHAPTER 7

Ex A

1) $5b^6$  2) $6c^7$  3) $b^3c^4$  4) $-12n^8$  5) $4n^5$  6) $d^2$  7) $a^6$  8) $-d^{12}$

Ex B

1) 2  2) 3  3) 1/3  4) 1/25  5) 1  6) 1/7  7) 9  8) 9/4  9) ¼  10) 0.2  11) 4/9  12) 64

13) $6a^3$ 14) $x$ 15) $xy^2$

**CHAPTER 8**

Ex A

1. $(x+2)^2-4$
2. $(x-3)^2-9$
3. $(x-8)^2-64$
4. $\left(x+\frac{1}{2}\right)^2-\frac{1}{4}$
5. $(x-7)^2-49$
6. $2(x+4)^2-32$
7. $3(x-4)^2-48$
8. $2(x-1)^2-2$
9. $5(x+2)^2-20$
10. $2\left(x-\frac{5}{4}\right)^2-\frac{25}{8}$
11. $3\left(x+\frac{3}{2}\right)^2-\frac{27}{4}$
12. $3\left(x-\frac{1}{2}\right)^2-\frac{1}{12}$

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## SOLUTIONS TO PRACTICE BOOKLET TEST

1) a) $4x^2+4x-3$ b) $a^2+6a+9$ c) $10x^2-13x$

2) a) $x(x-7)$ b) $(y+8)(y-8)$ c) $(2x-1)(x+3)$ d) $(3t-5)(2t-1)$

3) a) $\frac{x}{2y^2}$ b) $\frac{10x+3}{6}$

4) a) h = 5 b) $x=0$ or $x=8$ c) p = -6 or p = 2

5) a) $x^{-4}$ b) $x^6y^3$ c) $x^7$

6) a) $\frac{1}{16}$ b) 1 c) $\frac{2}{3}$

7) $x=3$, $y=4$

8) a) $x=\frac{v^2-u^2}{2a}$ b) $x=\sqrt{\frac{3V}{\pi h}}$ c) $x=\frac{2-y}{y-1}$

9) $x=\frac{1\pm\sqrt{21}}{10}$

10) $p=3$, $q=-5$

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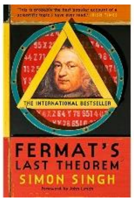
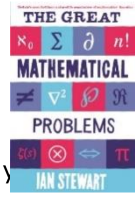
## A-Level Mathematics Suggested Reading List

The A-Level Mathematics Team at SMCS really want you to enjoy the beauty and rigor of Mathematics throughout your two years with us! Challenging yourself and exploring the subject outside of the curriculum is something we highly encourage , is a excellent habit to build and will support you in your future paths. Below are our recommendations for wider reading:




**Fermat's Last Theorem by Simon Singh**

The story of the solving of a puzzle that has confounded mathematicians since the 17th cent In 1963, a schoolboy browsing in his local library stumbled across the world's greatest mathematical problem: Fermat's Last Theorem, a puzzle that every child can understand but which has baffled mathematicians for over 300 years. Aged just ten, Andrew Wiles dreamed that he would crack it.
Wiles's lifelong obsession with a seemingly simple challenge set by a long-dead Frenchman is an emotional tale of sacrifice and extraordinary determination. In the end, Wiles was forced to work in secrecy and isolation for seven years, harnessing all the power of modern maths to achieve his childhood dream. Many before him had tried and failed, including a 18-century philanderer who was killed in a duel. An 18-century Frenchwoman made a major breakthrough in solving the riddle, but she had to attend maths lectures at the Ecole Polytechnique disguised as a man since women were forbidden entry to the school.

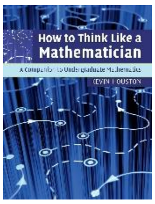



**The Great Mathematical Problems by Ian Stewart**

There are some mathematical problems whose significance goes beyond the ordinary - like Fermat's Last Theorem or Goldbach's Conjecture - they are the enigmas which define mathematics. This book explains why these problems exist, why they matter, what drives mathematicians to incredible lengths to solve them and where they stand in the context of mathematics and science as a whole. It contains solved problems - like the Poincaré Conjecture, cracked by the eccentric genius Grigori Perelman, who refused academic honours and a million-dollar prize for his work, and problems which, like the Riemann Hypothesis, remain baffling after centuries. Stewart is the guide to this mysterious and exciting world, showing how modern mathematicians constantly rise to the challenges set by their predecessors, as the great mathematical problems of the past succumb to the new techniques and ideas of the present.


**1089 and All That: A Journey into Mathematics by David Acheson**

David Acheson's extraordinary little book makes mathematics accessible to everyone. From very simple beginnings he takes us on a thrilling journey to some deep mathematical ideas. On the way, via Kepler and Newton, he explains what calculus really means, gives a brief history of pi, and even takes us to chaos theory and imaginary numbers. Every short chapter is carefully crafted to ensure that no one will get lost on the journey. Packed with puzzles and illustrated by world famous cartoonists, this is one of the most readable and imaginative books on mathematics ever written.




**How to Think Like a Mathematician by Kevin Houston**

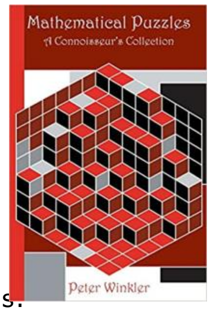
Looking for a head start in your undergraduate degree in mathematics? This friendly companion will ease your transition to real mathematical thinking. Working through the book you will develop an arsenal of techniques to help you unlock the meaning of definitions, theorems and proofs, solve problems, and write mathematics effectively. All the major methods of proof - direct method, cases, induction, contradiction and contrapositive - are featured. Concrete examples are used throughout, and you'll get plenty of practice on topics common to many courses such as divisors, Euclidean algorithms, modular arithmetic, equivalence relations, and injectivity and surjectivity of functions. With over 300 exercises to help you test your progress, you'll soon learn how to think like a mathematician.

Online Edition: 2-kevin-houston-how-to-think-like-a-mathematician.pdf (wordp


**Algorithmic Puzzles by Anany & Maria Levitin**

In this book, Anany and Maria Levitin use many classic brainteasers as well as newer examples from job interviews with major corporations to show readers how to apply analytical thinking to solve puzzles requiring well-defined procedures. The book's unique collection of puzzles is supplemented with carefully developed tutorials on algorithm design strategies and analysis techniques intended to walk the reader step-by-step through the various approaches to algorithmic problem solving. Mastery of these strategies - exhaustive search, backtracking, and divide-and-conquer, among others - will aid the reader in solving not only the puzzles contained in this book, but also others encountered in interviews, puzzle collections, and throughout everyday life. Each of the 150 puzzles contains hints and solutions, along with commentary on the puzzle's origins and solution methods. Readers with only middle school mathematics will develop their algorithmic problem-solving skills through puzzles at the elementary level, while seasoned puzzle solvers will enjoy the challenge of thinking through more difficult puzzles.

Online Edition: Algorithmic Puzzles (lagout.org)




**Mathematical Puzzles: A Connoisseur's Collection by Peter Winkler**

Collected over several years by Peter Winkler, dozens of elegant, intriguing challenges are presented in this book. The answers are easy to explain, but without this book, devilishly hard to find. Creative reasoning is the key to these puzzles. No involved computation or higher mathematics is necessary, but your ability to construct a mathematical proof will be severly tested - even if you are a professional mathematician. For the truly adventurous, there is even a chapter on unsolved puzzles.